VISCOSITY OF A SUSPENSION OF ELLIPSOIDAL FERROMAGNETIC PARTICLES IN A MAGNETIC FIELD

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The motion of a suspension of solid magnetized ellipsoids of rotation in a uniform magnetic field is considered. The ellipsoids are assumed to be magnetized along the axes of symmetry. Relaxation processes in the solid phase are not considered. The stress tensor of the suspension is calculated taking into account the rotational Brownian motion of the particles. It is shown that the viscosity tensor contains six independent kinetic coefficients, which are even with respect to the magnetic field. The relation between these coefficients and the field and the ratio of the semiaxes of the ellipsoid is obtained. As an example, the effect of the magnetic field on the symmetrical flow of the suspension in a contractile cylinder is considered.

An intensive experimental and theoretical investigation of suspensions of fine ferromagnetic particles (ferroliquids) has recently been made. The linear dimensions of the particles vary from 10^{-6} to 10^{-5} cm, while the magnetic moment μ may reach 10^4-10^5 Bohr magnetons. If the external magnetic field **H** is small compared with the internal anisotropy field, the magnetic moment can be assumed to be tightly bound to the particles (this problem is discussed in more detail in [1]). On the other hand, due to the high value of μ the dimensionless field $\xi = \mu H/kT$ may reach values of the order unity even when $H \sim 10^2$ Oe and at room temperature.

The volume density of the solid phase φ is assumed to be fairly small so that interaction between the particles can be neglected. The motion of the particles in the liquid is then governed by hydrodynamic forces, the orienting action of the field, and thermal fluctuations (the Brownian motion of the particles).

When there is no field the viscosity of the suspension increases as compared with the initial viscosity of the liquid. The magnetic field prevents free rotation of the particles in the vortex flow, which leads to additional, rotational, viscosity [2]. If the particles have a nonspherical shape, the field hinders their streamline flow even with a symmetrical flow. In this case, as shown below, the relation between the stress tensor of the suspension and the velocity gradient tensor is determined by six independent kinetic coefficients, which depend on the shape of the particles and on the magnetic field. Similar effects also occur in an external field in polar gases (the Zenftleben–Beenakker effect [3]). Although the nature of these phenomena in suspensions and gases are different, the equations of motion of a ferroliquid and a paramagnetic gas are almost identical, so that these systems behave in a similar manner in a magnetic field.

1. Regular Motion of a Magnetized Ellipsoid in a Flow. We will assume that the particles of the suspension are ellipsoids of rotation. The shape of the ellipsoids with semiaxes a and b (b is the radius of the circular cross section) can be represented by a dimensionless ratio s or by the nonsphericity parameter λ

$$s = a/b, \quad \lambda = (s^2 - 1)/(s^2 + 1) \tag{1.1}$$

We will describe the orientation of the ellipsoid by a single vector **e**, directed along its axis of symmetry. Assuming that the magnetic moment is strongly coupled to the particle, we have $\mathbf{m} = \mu\mathbf{e}$. The mean value of **m** defines the magnetization of the suspension of noninteracting ellipsoids

$$\mathbf{M} = \varphi \langle \mathbf{m} \rangle / V_1, \quad V_1 = 4\pi ab^2/3 \tag{1.2}$$

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A particle in a magnetic field is acted on by the moment of the forces

$$\mathbf{m} \times \mathbf{H} = kT\xi \mathbf{e} \times \mathbf{h} \tag{1.3}$$

(here $\mathbf{h}$ is the unit vector directed along the field). In an arbitrary flow of incompressible liquid $\mathbf{v}$ ($\mathbf{r}$) the ellipsoid of rotation is acted upon by the moment of the forces (see, for example [4, 5])

$$2\eta_0 V_1 S_1 [\Omega_i - \omega_i + \lambda e_{ikl} e_k e_m V_{lm}]$$
$$\Omega_{ik} = e_{ikl}\Omega_l = \frac{1}{2}\left(\frac{\partial v_k}{\partial x_i} - \frac{\partial v_i}{\partial x_k}\right), \quad V_{ik} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right), \quad V_{ii} = 0 \tag{1.4}$$

(ω is the angular velocity of a particle). The factor in front of the square brackets in Eq. (1.4) is the rotational mobility of the particles. Using the Einstein relation, we obtain the coefficient of rotational diffusion D

$$D = kT/4\eta_0 V_1 S_1 \tag{1.5}$$

The function $S_1(s)$, which defines the dependence of D on the ratio of the semiaxes of the ellipsoid s, is derived in the appendix.

2. Rational Diffusion. When the dimensions of the particles of the suspension are less than 10^{-4} cm, it is necessary to take their rotational Brownian motion into account. We will introduce the probability $W(\mathbf{e}, t)\, d\nu$ that the direction of the axis of a particle lies in the element of solid angle $d\nu$. The Brownian motion leads to an additional random moment of the forces acting on the particle [6]

$$-kTi\mathbf{N} \ln W \quad (\mathbf{N} = -i\mathbf{e} \times \partial/\partial \mathbf{e}) \tag{2.1}$$

For the rotational diffusion the operator of infinitely small rotation $\mathbf{N}$ plays the same role as the operator grad for translational diffusion. Note that $\mathbf{N}$ is a self-adjoint operator. The Fokker–Planck equation, which describes the rotational Brownian motion, can be written in the form

$$\partial W/\partial t + (i\mathbf{N}\omega) W = 0 \tag{2.2}$$

If we neglect the inertia of the particles, the angular velocity of their rotation ω is found by equating to zero the sum of the moments of the magnetic (1.3), hydrodynamic (1.4), and random (2.1) forces. This gives

$$\omega_i = D(e_{ikl} e_k \xi_l - iN_i \ln W) + \Omega_i + \lambda e_{ikl} e_k e_m V_{lm} \tag{2.3}$$

Equation (2.2) together with relation (2.3) plays the role of the kinetic equation for the ferromagnetic particles suspended in the liquid. In a stationary suspension the stationary solution of (2.2) and (2.3) is

$$W_0 = C \exp(\mathbf{e}\xi) \tag{2.4}$$

(The constant C is found from the normalization condition.)

The relaxation time of the distribution function W to the equilibrium value W_0 is of the order of 1/D. Taking the viscosity of the liquid as $\eta_0 \sim 10^{-2}$ g/cm · sec, $kT \sim 4 \cdot 10^{-14}$ erg, $V_1 \sim 10^{-18}$ cm^3, and assuming for $|\log s| \sim 1$, $S_1 \sim 10$, we obtain from Eq. (1.5), $1/D \sim 10^{-5}$ sec. This time is small compared with the hydrodynamic times $\rho l^2/\eta_0$ (l is the hydrodynamic scale of length), so that in fact we have to be concerned with the stationary solution of the Fokker–Planck equation. In addition, 1/D is also small compared with the velocity gradients. The condition $\Omega/D \ll 1$ is satisfied for all reasonable values of Ω. We will therefore seek the distribution function of the moving suspension in the form

$$W = W_0(1 + \chi) \tag{2.5}$$

where χ is a small correction, which is linear with respect to the velocity gradients Ω_{ik} and V_{ik}. In what follows it is convenient to introduce the following notation:

$$\langle A \rangle \equiv \int AW\, d\nu, \quad \langle A \rangle_0 = \int AW_0\, d\nu \tag{2.6}$$

The normalization condition leads to the following additional relation for the function χ: $\langle \chi \rangle_0 = 0$. Assuming the properties of spatial symmetry of χ, we can write

$$\chi = \chi_1 e_k h_i \Omega_{ik} + \chi_2 (e_i e_k - \langle e_i e_k \rangle_0) V_{ik} \tag{2.7}$$

where χ_1 and χ_2, generally speaking, are unknown functions of $\mathbf{e}\xi$ and ξ. Note that in what follows it will not be these functions which will be necessary but functionals of a form $\langle e_i \chi \rangle_0$.

To calculate χ_1 and χ_2 we can use approximate methods developed in the kinetic theory of gases [7]. Here it is convenient to use the Chapman–Enskog variational method. In the steady-state case Eqs. (2.2) and (2.3) can be rewritten in the form

$$iN_i\,(\Omega_i + \lambda e_{ikl}e_k e_m V_{lm})\,W = DiN_i\,(iN_i - e_{ikl}e_k\xi_l)\,W \tag{2.8}$$

The right-hand side of Eq. (2.8) plays the role of the "collision integral" and vanishes when $W = W_0$. In accordance with the chosen method, we assume χ_1 and χ_2 to be independent of e_i. Substituting Eqs. (2.5) and (2.7) into Eq. (2.8), multiplying the latter by e_i, and integrating over the angles, taking into account the Hermitian character of the operator N, and Eqs. (A. 4) and A.5), we obtain two equations which determine χ_1 and χ_2. As a result we have

$$\chi_1 = L_1\xi/D\,(\xi - L_1),\quad \chi_2 = \lambda/2\,D \tag{2.9}$$

where the functions $L_n(\xi)$ are determined in the appendix. Note that the value obtained for χ_2 is accurate.

Using the distribution function (2.5), and relations (1.2), (2.7), and (2.9), we will calculate the volume density of the moment of the forces $\mathbf{M}\times\mathbf{H}$, which acts on the moving suspension from the magnetic field side

$$\begin{gathered}(\mathbf{M}\times\mathbf{H})_i = -4\,[\eta_r\,(\Omega_i - h_i h_k\Omega_k) + \gamma e_{ikl}h_k h_m V_{lm}]\\ \eta_r = \eta_0\varphi S_1\xi L_1^2/(\xi - L_1),\quad \gamma = \eta_0\varphi\lambda S_1 L_2\end{gathered} \tag{2.10}$$

The meaning of the coefficients η_r and γ will become clear in the following section.

3. The Stress Tensor. The stress tensor of the suspension Σ_{ik} is best divided into two terms, by separating from it the Maxwell tensor π_{ik}

$$\Sigma_{ik} = \sigma_{ik} + \pi_{ik},\quad \pi_{ik} = (H_i B_k - H^2\delta_{ik}/2)/4\,\pi \tag{3.1}$$

The tensor σ_{ik} is found from the solution of the auxiliary hydrodynamic problem of the perturbation of the flow of liquid by the moving ellipsoid [4, 5]. After averaging the perturbed stress tensor and the velocity gradient tensor over the spatial coordinates, and over the orientations of the ellipsoid, we can write

$$\sigma_{ik} = \sigma^\circ\delta_{ik} + e_{ikl}\sigma_l^a + \sigma_{ik}^s \tag{3.2}$$

where the scalar σ°, the vector σ_l^a, and the symmetrical tensor $\sigma_{ik}^s\,(\sigma_{ii}=0)$ have the form

$$\sigma^\circ = -p + \eta_0\varphi k_1\langle e_l e_m\rangle\,V_{lm} \tag{3.3}$$

$$\sigma^a = -{}^1/_2\mathbf{M}\times\mathbf{H} \tag{3.4}$$

$$\begin{gathered}\sigma_{ik}^s = 2\eta_0\,[(1+\varphi k_0)\,V_{ik} + \varphi\,(k_2\langle e_l e_m\delta_{kl} + e_k e_m\delta_{il}\rangle +\\ + k_3\langle e_l e_m\rangle\delta_{ik} + k_4\langle e_i e_k e_l e_m\rangle)\,V_{lm} + \varphi k_5\langle e_m e_k\delta_{il} + e_i e_m\delta_{kl}\rangle\,\Omega_{lm}]\end{gathered} \tag{3.5}$$

In these expressions we mean by V_{ik} and Ω_{ik} the quantities (1.4), obtained by averaging, which we mentioned above. The coefficients k_n depend on s. When deriving relations (3.3)-(3.5) we used Eq. (2.8).

Although σ_{ik} contains the nonsymmetrical part (3.4), the complete stress tensor Σ_{ik} taking into account the Maxwell tensor π_{ik} is symmetrical.

When calculating the averages which occur in Eqs. (3.3) and (3.5) in the approximation which is linear with respect to Ω_{ik} and V_{ik}, it is sufficient to confine ourselves to the functions W_0. In Eq. (3.4), it is necessary to take into account the perturbation of the distribution function due to the motion of the liquid (see Section 2). Using Eqs. (A.4)-(A.6) and relation (2.10), we obtain

$$\sigma^\circ = -p + 3\,\beta h_i h_k V_{ik} \tag{3.6}$$

$$\sigma_i^a = 2\,\eta_r\,(\Omega_i - h_i h_k\Omega_k) + 2\,\gamma e_{ikl}h_k h_m V_{lm} \tag{3.7}$$

$$\begin{gathered}\sigma_{ik}^s = 2(2\eta_2 - \eta_1)\,V_{ik} + 2\,[(\eta_2 - \eta_1)h_l h_m\delta_{ik} + (\eta_1 + \eta_3 -\\ - 2\eta_2)(h_i\delta_{mk} + h_k\delta_{im})h_l + (\eta_1 + \eta_2 - 2\eta_3)h_i h_k h_l h_m]V_{lm} + 2\gamma(h_i\delta_{mk} + h_k\delta_{im})h_l\Omega_{lm}\end{gathered} \tag{3.8}$$

The kinetic coefficients η_n (n = 1, 2, 3), β, γ, and η_r which occur here depend upon ξ and s

$$\beta = \eta_0\varphi S_3 L_2 \tag{3.9}$$

while in the coefficients η_n it is convenient to separate the part which is independent of the field

$$\eta_n = \eta_0 + \Delta\eta_0 + \Delta\eta_n \quad (n = 1,\ 2,\ 3) \tag{3.10}$$

where

$$\Delta\eta_0 = \eta_0\varphi S_2, \ \Delta\eta_1 = \eta_0\varphi\,(S_3L_2 + S_4L_3/\xi)$$
$$\Delta\eta_2 = \eta_0\varphi S_5L_4, \ \Delta\eta_3 = \eta_0\varphi\,(S_6L_2 + S_7L_3/\xi) \tag{3.11}$$

The functions $L_n(\xi)$ and $S_n(s)$ are determined in the appendix.

To explain the meaning of the kinetic coefficients introduced above it is necessary to write the stress tensor (3.6)-(3.8) for the case of a field **H** directed along the x axis. Then, comparison with the phenomenological equations in [8] shows that the quantity η_r is the rotational viscosity of the suspension, since it connects the corresponding components σ_i^a and Ω_i. Note that the component Ω, parallel to **H**, makes no contribution to the stress tensor, since in this case the magnetic field does not hinder the rotation of the particles, and additional energy dissipation does not occur.

The quantities η_n have the meaning of shear viscosities (compare with [8]).

The coefficient β is a cross between the shear and volume effects of the viscous friction, while γ is a cross between the effects of the rotational and shear frictions. The Onsager symmetry principle is satisfied for the cross coefficients.

All the kinetic coefficients are even functions of the magnetic field. The absence of odd effects is due to the specific features of the chosen model (the fact that **e** and **m** are parallel).

If the field **H** =0, the stress tensor of the suspension takes the form

$$\Sigma_{ik} = -p\delta_{ik} + 2\,(\eta_0 + \Delta\eta_0)\ V_{ik}$$

i.e., $\Delta\eta_0$ determines the increase in the viscosity of the suspension compared with the initial viscosity of the liquid. For spherical particles (s =1), $S_2 = 5/2$, and Eq. (3.11) gives the well-known Einstein result.

Using the expansion in series of the functions $L_n(\xi)$ (Sec. 3), and expressions (2.10), (3.9), and (3.11), it can be seen that for small fields ($\xi \ll 1$), $\Delta\eta_2 \sim \xi^4$, while the remaining coefficients increase as the square of the field. Since $L_n \to 1$, when $\xi \gg 1$, we obtain that in intense fields all the coefficients cease to depend on the field. In this case $\beta, \gamma,\ \eta_r$ and $\Delta\,\eta_2$ saturate according to the same law for all values of s. In Fig. 2 the curves show the ratio of the kinetic coefficients to their values when $\xi \to \infty$ as a function of the dimensionless magnetic field ξ: 1) $\Delta\eta_2/\Delta\eta_2(\infty) = L_4$, 2) $\Delta\eta_1/\Delta\eta_1(\infty)$ for s =0.1, and $\Delta\eta_3/\Delta\eta_3(\infty)$ for s =10, 3) $\gamma/\gamma(\infty) = \beta/\beta(\infty) = L_2$, 4) $\Delta\eta_1/\Delta\eta_1(\infty)$ for s =10, 5) $\eta_r/\eta_r(\infty)$, 6) $\Delta\eta_3/\Delta\eta_3(\infty)$ for s =0.1. For large (small) values of s, $\Delta\eta_1$ ($\Delta\eta_3$) saturates somewhat earlier than for small (large) values of s.

In Fig. 1 the curves show the kinetic coefficients as a function of the ratio of the semiaxes of the ellipsoid s for saturation ($\xi \to \infty$): 1) $\eta_r(\infty)/\eta_0\varphi = S_1$, 2) $\gamma(\infty)/\eta_0\varphi = \lambda S_1$, 3) $\Delta\eta_0/\eta_0\varphi = S_2$, 4) $\Delta\eta_1(\infty)/\eta_0\varphi = S_3$, 5) $\Delta\eta_3(\infty)/\eta_0\varphi = S_6$, 6) $\beta(\infty)/\eta_0\varphi = S_8$, 7) $\Delta\eta_2(\infty)/\eta_0\varphi = S_5$. Note that when changing from oblate particles (s < 1) to elongated particles (s > 1), $\Delta\eta_1, \Delta\eta_3, \gamma, \beta$ change sign; $\Delta\eta_2$ is always negative while η_r is always positive. For spherical particles only the rotational viscosity differs from zero. Using Eqs. (A.9)-(A.11) we can explain the dependence of the kinetic coefficients on s in different limiting cases.

4. Discussion. Since in a uniform magnetic field $(\partial\pi_{ik}/\partial x_k) = 0$, only the part of the stress tensor σ_{ik} makes a contribution to the equation of motion of the liquid

$$\rho\partial v_i/\partial t = \partial\sigma_{ik}/\partial x_k \tag{4.1}$$

Substituting Eqs. (3.2), (3.6)-(3.8) into Eq. (4.1) and taking into account the other equations of hydrodynamics, which have the usual form, we obtain the complete system of hydrodynamic equations of the ferrosuspension in a magnetic field (with the usual boundary conditions for the velocity and the field).

In order to establish the possibility of experimentally measuring the kinetic coefficients, we must consider specific examples of the flow of a ferroliquid.

The motion of a suspension of spherical particles in a circular capillary was considered in [2], and the part played by the rotational viscosity was clarified.

We note further that Eq. (4.1) with the stress tensor (3.6)-(3.8) has the same form as in the case of a paramagnetic gas (if we put $\eta_r = \gamma = 0$). Plane Poiseuille flow in a magnetic field is considered for a gas

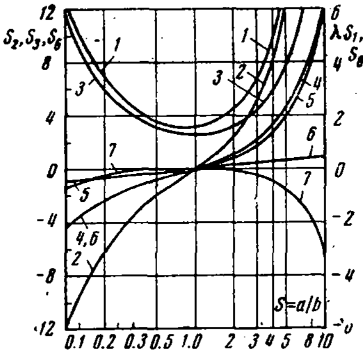

Fig. 1

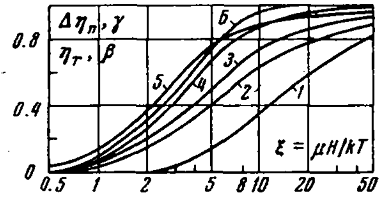

Fig. 2

in [8], and it is shown that all the viscosity coefficients can be obtained from measurements of the flow rate and the transverse pressure gradient in a rectangular capillary for different orientations of the field. It is obvious that these results can be transferred to the case of a ferrosuspension bearing in mind the small changes due to the presence in σ_{ik} of terms proportional to η_r and γ. (As can be shown, using perturbation theory developed in [10], in a gas a coefficient of the type γ should be proportional to $\lambda^2 \xi^2$. Since for gas molecules $\lambda \sim 1/5$, while μ is of the order of the Bohr magneton, for all reasonable temperatures γ is negligibly small for gases.)

To clarify the part played by the coefficient γ, we will calculate the moment of the force K which acts on a ferroliquid for uniaxial symmetrical flow of the form

$$\mathbf{v} = \alpha(\mathbf{r}/3 - \mathbf{k}z) \tag{4.2}$$

(the unit vector k is directed along the z axis). This type of flow occurs, for example, if the liquid is inside a cylinder, on the base of which constant forces act which lead to compression of the cylinder at constant volume.

To calculate K we integrate (2.10) over the volume of the cylinder V. In this case to a first approximation with respect to φ it is sufficient to confine ourselves to the unperturbed motion (4.2). Choosing the x axis in the kh plane, and denoting by θ the angle between the axis of the cylinder k and the direction of the field, we obtain

$$\mathbf{K} = 4\alpha\gamma V(\mathbf{hk})\,\mathbf{h}\times\mathbf{k},\quad K_x = K_z = 0,\quad K_y = 2\alpha\gamma V\sin 2\theta \tag{4.3}$$

As can be seen from Eqs. (4.3), the moment of the forces acting on the cylinder arises if simultaneously $h_x \neq 0$, and $h_z \neq 0$, and reaches a maximum value when $h_x = h_z$ ($\theta = 45°$). It is interesting to note that the moment of the forces depends very much on the shape of the particles, and changes sign when the particles change from being oblate to elongated.

In conclusion I wish to thank M. I. Shliomis, and Yu. L. Raikher for useful discussion.

Appendix. The moments of the distribution function W_0 from Eq. (2.4) can be expressed in terms of the function

$$L_n(\xi) = I_{n+1/2} / I_{1/2} \tag{A.1}$$

where $I_{n+1/2}(\xi)$ is the Bessel function of imaginary argument. It follows from (A.1) that $L_0 = 1$ while L_1 is identical with the Langevin function

$$L_1 = L = \operatorname{cth}\xi - 1/\xi \tag{A.2}$$

The functions L_n with $n > 1$ can be obtained from the recurrence relations for Bessel functions: $L_{n-1} - L_{n+1} = (2n+1)\,L_n/\xi$.

If $\xi \to \infty$, we have $L_n = 1 + O(1/\xi)$, and when $\xi \to 0$

$$L_n = \frac{\xi^n}{(2n+1)!!}\left(1 - \frac{n}{3(2n+3)}\xi^2 + O(\xi^4)\right) \tag{A.3}$$

We will now derive the moments of the function W_0

$$\langle e_i\rangle_0 = L_1 h_i,\quad \langle e_i e_k\rangle_0 = (L_1/\xi)\delta_{ik} + L_2 h_i h_k \tag{A.4}$$

$$\langle e_i e_k e_l \rangle_0 = (L_2 / \xi)(h_i \delta_{kl} + h_k \delta_{il} + h_l \delta_{ik}) + L_3 h_i h_k h_l \tag{A.5}$$

$$\langle e_i e_k e_l e_m \rangle_0 = (L_1 / \xi^2)(\delta_{ik}\delta_{lm} + \delta_{im}\delta_{kl} + \delta_{il}\delta_{km}) + (L_3 / \xi)(h_i h_k \delta_{lm} + h_i h_m \delta_{kl} + \ldots) + L_4 h_i h_k h_l h_m \tag{A.6}$$

The function $S_n(s)$, which determines the dependence of the kinetic coefficients on the ratio of the semiaxes of the ellipsoid, can be represented in the form of linear combinations of the functions f_m

$$S_n = \sum_{m=1}^{5} a_{nm} f_m \qquad (n = 1, 2, \ldots, 8) \tag{A.7}$$

$$\begin{gathered} f_1 = 10 s f_4 (2s - J) / 9 (s^2 - 1), \quad f_2 = 4 (s^2 - 1)^2 / 9s (4s^3 - 10s + 3J) \\ f_3 = 4\lambda (s^2 - 1) / 3 (2s^2 + 4 - 3sJ), \quad f_4 = 2 (s^2 - 1)^2 / 9s (2s^2 J + J - 6s) \\ f_5 = 4\lambda (s^2 - 1)^2 / s [(2s^2 - 1) J - 2s], \qquad J = \int_0^\infty dx / (1 + x)(s^2 + x)^{1/2} \end{gathered} \tag{A.8}$$

while the coefficients a_{nm} can be written in the form of the matrix

$$a_{mn} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 12 & -6 & -36 & 3 & -12 & 24 & 10 \\ 0 & 2 & -2 & 24 & -2 & 3 & -16 & 0 \\ 0 & 2 & 8 & -36 & 3 & -2 & 24 & -10 \\ \frac{1}{2}\lambda^2 & 2 & -2 & 12 & -2 & 3 & -16 & 0 \end{pmatrix}$$

These equations can be simplified considerably in the limiting cases of very flattened, elongated, and close to spherical particles. For $s \ll 1$ we have

$$S_n = 2a_n / 9\pi s \tag{A.9}$$

where the numerical coefficients a_n ($n = 1.2, \ldots, 8$) can be written in the form of a row $a_n = (9\ 16\ -6\ 24\ -1\ -1\ -8\ -^{10}/_3)$.

If $s \gg 1$, we have $S_8 = {}^2/_3$, while for the other functions S_n we obtain

$$S_n = b_n s^2 / 9 \ln s \qquad (n \neq 8;\ b_n = 4.5\ 60\ 5\ 18\ -1.5\ 7.5\ -12) \tag{A.10}$$

For particles which are almost spherical ($\lambda \to 0$), $S_n = 0(\lambda^2)$ for $n = 4, 5, 7$; for other values of n we have

$$\begin{gathered} S_1 = \frac{3}{2} - \frac{6}{5}\lambda, \qquad S_2 = \frac{5}{2} + 0(\lambda^2), \\ S_3 = \frac{25}{21}\lambda, \qquad S_6 = \frac{25}{42}\lambda, \qquad S_8 = \frac{20}{63}\lambda \end{gathered} \tag{A.11}$$

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